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THE BASE, SUB-BASE, AND QUASI-NEIGHBORHOOD IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

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Abstract:

Fuzzy topology was developed by many authors. The intuitionistic fuzzy sets were introduced by Atanassov (1986) as a generalization of fuzzy sets by L. A. Zadeh (1965).

The paper is therefore addressed to the introduction of the concept of base, sub-base, subspace, closure, interior and quasi neighbourhood in intuitionistic fuzzy topology developed by Kumar and Das (2019).

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, fuzzy point, Q-neighbourhood.

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1. Introduction.

Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets of L. A. Zadeh [1], where besides the degree of membership $\mu_A(x) \in [0,1]$ for each $x \in X$ to a set A, the degree of non-membership $\mu'_A(x) \in [0,1]$ was also considered.

The intuitionistic fuzzy set (IFS) is a sufficiently generalisation to include both fuzzy sets and vague sets. The study of weaker forms of different notions of intuitionistic fuzzy topology is currently under way [3, 4]. Kumar and Das [11] developed an intuitionistic fuzzy topological space in which the concepts of continuity and semi-regular closed sets have been discussed.

In this paper, we extend this topology which will contain base, sub base, subspace, fuzzy point, closure, interior and neighbourhood with various properties and examples. The concept of fuzzy topology by Palaniappan [5] proved to be the basic tool for development.

2. Preliminaries.

Definition (2.1) : A fuzzy set $\tilde{A}$ on a non empty set X is characterised by its membership function $\mu_{\tilde{A}}(x) \in [0,1]$, where $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of elements of X in fuzzy set $\tilde{A}$ for each $x \in X$ [1].

Definition (2.2) : A family $\delta \subseteq I^{*}(I = [0,1])$ is called a fuzzy topology on X if it satisfies the following conditions-

(i) $\forall \ \alpha \in I$, $\alpha \in \delta$

(ii) $\forall \ A, \ B \in \delta \Rightarrow A \wedge B \in \delta$

(iii) $\forall \ (Aj)_{j \in J} \in \delta \Rightarrow \underset{j \in J}{V} Aj \in \delta$

The pair (X, δ) is called a fuzzy topological space. The elements of δ are called fuzzy open sets. A fuzzy set K is called fuzzy closed set if $K^{c} \in \delta$.

We write by δ^{c}, the collection of all fuzzy closed sets in this fuzzy topological space

Obviously then

(i) $\alpha^{c} \in \delta^{c}$

(ii) $K, \ M \in \delta^{c} \Rightarrow K \vee M \in \delta^{c}$

(iii) $\{Kj : j \in J\} \in \delta^{c} \Rightarrow \wedge \{Kj, \ j \in J\} \in \delta^{c}$ [5]

Definition (2.3) : An intuitionistic fuzzy set (IFS) $\tilde{A}$ on X is defined as the set of all ordered triplets of the form $\tilde{A} = \{(x, \ \mu_{\tilde{A}}(x), \ \mu'_{\tilde{A}}(x) \ )\}, \quad x \in X$

Where the function $\mu_{\tilde{A}} : X \rightarrow [0,1]$ and $\mu'_{\tilde{A}} : X \rightarrow [0,1]$ define the degree of membership and degree of non-membership of the elements $x \in X$ respectively and for every $x \in X$ in $\tilde{A}$, $0 \le \mu_{\tilde{A}}(x) \ + \mu'_{\tilde{A}}(x) \le 1$.

Example (2.4) : Let X = {a, b, c, d}, then $\tilde{A}$ = { (a, .1, .9), (b, 0, .9), (c, .2, .7), (d, .3, .6) } is an intuitionistic fuzzy set (IFS). The support of an IFS $\tilde{A}$ on X is the crisp set of all $x \in X$ with non-zero membership grade.

In this example $\text{Supp}(\tilde{A}) = \{a, c, d\}$, The family of all intuitionistic fuzzy sets on X is denoted by IFS (X) [2].

Definition (2.5): Let X be a non empty set and

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \mu'_{\tilde{A}}(x)\}$, $\tilde{B} = \{(x, \mu_{\tilde{B}}(x), \mu'_{\tilde{B}}(x)\}$ be two IFS in X,

Then (i) $\tilde{A} \subseteq \tilde{B}$ *if* $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$ and $\mu'_{\tilde{A}}(x) \ge \mu'_{\tilde{B}}(x)$

(ii) $(\tilde{A})^C = \{(x, \mu'_{\tilde{A}}(x), \mu_{\tilde{A}}(x)) : x \in X\}$ is complement of $\tilde{A}$

(iii) $\tilde{A} \cup \tilde{B} = \{(x, \mu_{\tilde{A}}(x) \vee \mu_{\tilde{B}}(x), \mu'_{\tilde{A}}(x) \wedge \mu'_{\tilde{B}}(x))\}$

(iv) $\tilde{A} \cap \tilde{B} = \{(x, \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x), \mu'_{\tilde{A}}(x) \vee \mu'_{\tilde{B}}(x))\}$ [4]

Definition (2.6) : Let $\tilde{B} = \{(y, \mu_{\tilde{B}}(y), \mu'_{\tilde{B}}(y)\}$ be an IFS in Y, Then The pre-image of $\tilde{B}$ under f denoted by $f^{-1}(\tilde{B})$ is the IFS in X defined by $f^{-1}(\tilde{B}) = \{(x, f^{-1}(\mu_{\tilde{B}}(x)), f^{-1}(\mu'_{\tilde{B}}(x))\}$

If $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \mu'_{\tilde{A}}(x))\}$ be an IFS in X , Then the image of $\tilde{A}$ under f denoted by $f(\tilde{A})$ is the IFS in Y defined by $f(\tilde{A}) = \{(y, f(\mu_{\tilde{A}}(y), f(\mu'_{\tilde{A}}(y))\}$

$$\text{Where } f(\mu_{\tilde{A}}(y)) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_{\tilde{A}}(x) & \text{if } \quad f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

$$\text{And } f(\mu'_{\tilde{A}}(y)) = \begin{cases} \inf \mu'_{\tilde{A}}(x) & \text{if } \quad f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases} \qquad [6, 7, 8]$$

3. Intuitionistic Fuzzy Topological Space.

An intuitionistic fuzzy topology (IFT) on a non empty set X is a family τ of IFSs in X satisfying the following axioms

τ_1 : $\tilde{0}, \tilde{1} \in \tau$

τ_2 : $\tilde{G}_1 \cap \tilde{G}_2 \in \tau$ for any $\tilde{G}_1, \tilde{G}_2 \in \tau$

τ_3 : $\cup G_i \in \tau$ for any arbitrary family $\{ G_i : i \in \Delta \} \subseteq \tau$

Where $\tilde{0} = \{ (x, 0, 1) : x \in X \}$

$\tilde{1} = \{ (x, 1, 0) : x \in X \}$

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and members of τ are called intuitionistic fuzzy open sets.

An IFS $\tilde{G}$ is called intuitionistic fuzzy closed set if $\tilde{G}^C \in \tau$ [11].

We denote by τ^c, the collection of all intuitionistic fuzzy closed sets in this IFTS Obviously then,

(i) $\tilde{1}, \tilde{0} \in \tau^c$

(ii) $\tilde{A}_1, \tilde{A}_2 \in \tau^c \Rightarrow \tilde{A}_1 \cup \tilde{A}_2 \in \tau^c$

(iii) $\{\tilde{A}j, j \in \Delta \} \in \tau^c \Rightarrow \wedge \{\tilde{A}j, j \in \Delta \} \in \tau^c$

Example (3.1) : Let X = {a, b} and $\tilde{G}$ be an IFS in X given by $\tilde{G}$ = { (a, 0.5, 0.4), (b, 0.4, 0.3) } then $\tau = \{ \tilde{0}, \tilde{G}, \tilde{1} \}$ is an IFT.

Example (3.2) : Let X = {a, b } and $\tilde{G}_1$, $\tilde{G}_2$ be IFSs on X given by

$\tilde{G}_1$ = { (a, .3, .6), (b, .1, .7) }

$\tilde{G}_2$ = { (a, .5, .3), (b, .6, .4) }

Then $\tau = \{ \tilde{0}, \tilde{G}_1, \tilde{G}_2, \tilde{1} \}$ is an IFT on X

Because $\tilde{G}_1 \cup \tilde{G}_2$ = { (a, max (.3, .5), min (.6, .3)), (b, max (.1, .6), min (.7, .4)) }

= { (a, .5, .3), (b, .6, .4) } = $\tilde{G}_2 \in \tau$

Similarly $\tilde{G}_1 \cap \tilde{G}_2 = \tilde{G}_1 \in \tau$

$\tilde{1} \cup \tilde{G}_1$ = { (a, 1, 0), (b, 1, 0) } $\cup$ { (a, .3, .6), (b, .1, .7) }

= { (a, 1, 0), (b, 1, 0) } = $\tilde{1} \in \tau$

$\tilde{1} \cap \tilde{G}_1$ = { (a, .3, .6), (b, .1, .7) } = $\tilde{G}_1 \in \tau$

On the same way $\tilde{1} \cup \tilde{G}_2$, $\tilde{1} \cap \tilde{G}_2$, $\tilde{0} \cup \tilde{G}_1$, $\tilde{0} \cap \tilde{G}_1$ all are in τ .

Example (3.3) : Let X = { a, b } and $\tilde{G}_1$ = { (a, .4, .3), (b, .3, .1) } , $\tilde{G}_2$ = { (a, .5, .2), (b, .4, .3) }

be two IFSs, then $\tau = \{ \tilde{0}, \tilde{G}_1, \tilde{G}_2, \tilde{1} \}$ is not an IFT because $\tilde{G}_1 \cup \tilde{G}_2 \notin \tau$ and $\tilde{G}_1 \cap \tilde{G}_2 \notin IFS(\text{X})$.

4. Base, Sub-Base, Sub-Space, Fuzzy Point :

Definition (4.1) : A collection β of IFSs is called a base for an IFTS (X, τ) if

(i) $\beta \subset \tau$

(ii) $\tilde{G} \in \tau \Rightarrow \tilde{G} = \vee_{j \in \Delta} \text{A}j \quad \textit{for some} \quad \tilde{\text{A}}j \in \beta$

Example (4.2) Let X = { a, b, c, d } and

$\tau = \left\{ \tilde{0}, \tilde{1}, \tilde{G}_1, \tilde{G}_2, \tilde{G}_1 \cup \tilde{G}_2, \tilde{G}_3 \cup \tilde{G}_4, \tilde{G}_1 \cup \tilde{G}_3 \cup \tilde{G}_4, \tilde{G}_2 \cup \tilde{G}_3 \cup \tilde{G}_4 \right\}$ be an IFT on X

Where $\tilde{G}_1$ = { (a, .1, .9), (b, .2, .6), (c, .3, .7), (d, .5, .3) }

$\tilde{G}_2$ = { (a, .2, .5), (b, .3, .5), (c, .4, .6), (d, .3, .4) }

$\tilde{G}_3$ = { (a, .1, .2), (b, .2, .3), (c, .4, .1), (d, .5, .1) }

$\tilde{G}_4$ = { (a, .3, .5), (b, .6, .2), (c, .7, .2), (d, .8, .1) }

Then $\beta = \left\{ \tilde{G}_1, \tilde{G}_2, \tilde{G}_3 \cup \tilde{G}_4 \right\}$ is a base for τ

Definition (4.3) :A collection β^* of IFSs is called a sub-base for the IFTS (X, τ) if

(i) $\beta^* \subset \tau$

(ii) The collection of infimum of finite sub families of β^* forms a base for (X, τ).

Example (4.4) : Let X = { a, b, c, d } and

$\tau = \left\{ \tilde{0}, \tilde{1}, \tilde{G}_1, \tilde{G}_1 \cup \tilde{G}_3, \tilde{G}_1 \cup \tilde{G}_4, \tilde{G}_1 \cup \tilde{G}_3 \cup \tilde{G}_4 \right\}$ be an IFT on X, Then

$\beta^* = \left\{ \tilde{G}_1 \cup \tilde{G}_4 , \tilde{G}_1 \cup \tilde{G}_3 \right\}$ is sub-base for τ

Where $\tilde{G}_1$, $\tilde{G}_2$, $\tilde{G}_3$, $\tilde{G}_4$ are same as in example (4.2).

Definition (4.5) :Let (X, τ) be an IFTS and $Y \subset X$. Then the collection $u = \{ \tilde{A} \setminus Y : \tilde{A} \in \tau \}$ where $\tilde{A} \setminus Y = \{ y, \ \mu_{\tilde{A} \setminus Y}(y), \ \mu'_{\tilde{A} \setminus Y}(y) \ ) : y \in Y \}$ is also an IFT on Y called relativised IFT and (Y, u) is called a sub-space of (X, τ).

Example (4.6) : Let X = { a, b, c } and $\tau = \{ \tilde{0}, \ \tilde{G}_1, \ \tilde{G}_2, \ \tilde{1} \}$ be an IFT on X,

where $\tilde{G}_1$ = { (a, .4, .5), (b, .3, .7), (c, .1, .8) }

$\tilde{G}_2$ = { (a, .6, .4), (b, .3, .4), (c, .2, .4) }

Let $Y = \{ a, b \} \subset X$

Representing Y as IFS, we have

$\tilde{Y}$ = { (a, 1, 0), (b, 1, 0), (c, 0, .5) }

Obviously $\tilde{G}_1 \cap \tilde{Y}$ = { (a, .4, .5), (b, .3, .7), (c, 0, .8) }

$\tilde{G}_2 \cap \tilde{Y}$ = { (a, .6, .4), (b, .3, .4), (c, 0, .5) }

We write $\tilde{G}_1 \cap \tilde{Y} = \tilde{G}_1^*$, $\tilde{G}_2 \cap \tilde{Y} = \tilde{G}_2^*$

Then $u = \{ \tilde{0}, \ \tilde{G}_1^*, \ \tilde{G}_2^*, \ \tilde{1} \}$ is an IFT on Y

$\Rightarrow (Y, u)$ is a subspace of (X, τ)

Definition (4.7) : An intuitionistic fuzzy point (IFP) P of X is a special intuitionistic fuzzy set in X which takes the value (0, 1) for all $y \in X$ except one say $x \in X$ and is denoted by

$$P\left[x_{(\alpha,\beta)}{}^{(y)}\right] = \begin{cases} (\alpha,\beta) & if \quad y = x \\ (0,1) & if \quad y \neq x \end{cases}$$

x is called the support of $x_{(\alpha,\beta)}$; α *and* β being value and non-value of $x_{(\alpha,\beta)}$, where

$\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$.

An intuitionistic fuzzy point (IFP) $x_{(\alpha,\beta)}$ will belong to $\tilde{A} = \{ (\ x, \ \mu(x), \ \mu'(x) \ ) \}$ in X denoted by $x_{(\alpha,\beta)} \in \tilde{A}$ if

$$\alpha \leq \mu(x) \quad and \quad \beta \geq \mu'\ (x)$$

The family of all IFPs in X is denoted by IFP (X)

In short, we shall denote IFP by $P_x^{(\alpha,\beta)}$ whose complement will be $P_x^{1-\alpha,\ 1-\beta}$ or $\left[P_x^{(\alpha,\beta)}\right]^C$.

Two IFSs $\tilde{A}$ and $\tilde{B}$ in X will be said to intersect if $\exists$ a point $x \in X$ such that $\mu_{\tilde{A}\cap\tilde{B}}(x) \neq 0$ and $\mu'_{\tilde{A}\cap\tilde{B}}(x) \neq 1$

Proposition (4.8) : Let $\left\{\tilde{A}j : j \in J\right\}$ be a family of IFSs in X, $P_x^{(\alpha,\beta)}$ and $P_y^{(\gamma,\delta)}$ be two IFPs in X. Then

(i) $P_x^{(\alpha,\beta)} \in \vee\left\{\ \tilde{A}j,\ \ j \in J\ \right\} \Leftrightarrow \exists\ j \in J$ such that $P_x^{(\alpha,\beta)} \in \tilde{A}j$

(ii) If $P_x^{(\alpha,\beta)} \in \wedge\left\{\ \tilde{A}j,\ \ j \in J\ \right\}$, then $\forall\ \ j \in J$, $P_x^{(\alpha,\beta)} \in \tilde{A}j$

(iii) If $P_x^{(\alpha,\beta)} \in P_y^{(\gamma,\delta)}$ and for every $j \in J$, $P_y^{(\gamma,\delta)} \in \tilde{A}j$

Then $P_x^{(\alpha,\beta)} \in \wedge\left\{\ \tilde{A}j,\ \ j \in J\ \right\}$

Proof (i) : Let $P_x^{(\alpha,\beta)} \in \tilde{A}j$

$$\Rightarrow \alpha \leq \mu_{\tilde{A}j}(x) \quad and \quad \beta \geq \mu'_{\tilde{A}j}(x)$$

$$\Rightarrow \alpha \leq \max\left\{\ \mu_{\tilde{A}j}(x)\ \ :\ \ j \in J\ \right\}$$

and $\beta \geq \max\left\{\ \mu'\ _{\tilde{A}j}(x)\ \ :\ \ j \in J\ \right\}$

$$\Rightarrow \alpha \leq \mu_{(\vee\tilde{A}j)}(x) \quad and \quad \beta \geq \mu'_{(\vee\tilde{A}j)}(x)$$

$$\Rightarrow P_x^{(\alpha,\beta)} \in \vee\left\{\ \tilde{A}j,\ \ j \in J\ \right\}$$

Conversely let $P_x^{(\alpha,\beta)} \in \vee\left\{\ \tilde{A}j,\ \ j \in J\ \right\}$

$$\Rightarrow \alpha \leq \mu_{(\vee\tilde{A}j)}(x)\ ,\quad \beta \geq \mu'\ _{(\vee\tilde{A}j)}(x)$$

$$\Rightarrow \alpha \leq \mu_{\tilde{A}j}(x)\ ,\quad \beta \geq \mu'\ _{\tilde{A}j}(x)$$

$$\Rightarrow P_x^{(\alpha,\beta)} \in \tilde{A}j$$

(ii) obvious

(iii) Given $P_x^{(\alpha,\beta)} \in P_y^{(\gamma,\delta)}$ *and* $P_y^{(\gamma,\delta)} \in \tilde{A}j$

$$\Rightarrow P_x^{(\alpha,\beta)} \in \tilde{A}j$$

$$\Rightarrow \alpha \le \mu_{\tilde{A}j}(x), \quad \beta \ge \mu'_{\tilde{A}j}(x)$$

$$\Rightarrow \alpha \le \min\left\{ \mu_{\tilde{A}j}(x)\right\}, \ \beta \ge \min\left\{ \mu'_{\tilde{A}j}(x)\right\}$$

$$\Rightarrow \alpha \le \mu_{(\wedge\tilde{A}j)}(x) \ , \ \beta \ge \mu'_{(\wedge\tilde{A}j)}(x)$$

$$\Rightarrow P_x^{(\alpha,\beta)} \in \wedge\left\{ \tilde{A}j \ : \ j \in J \right\}$$

Proposition (4.9) : Let $P_x^{(\alpha,\beta)}$ be IFP in X and $f : x \to y$ is a mapping, Then

(i) $f\left(P_x^{(\alpha,\beta)}\right) = P_{f(x)}^{(\alpha,\beta)}$

(ii) $f\left\{ \left(P_x^{(\alpha,\beta)}\right)^C \right\} = \left\{ f\left(P_x^{(\alpha,\beta)} \right) \right\}^C$

Proof : (i) we have

$$f\left(P_x^{(\alpha,\beta)}\right)(y) = \begin{bmatrix} \sup\limits_{x\in f^{-1}(y)} \mu_{P_x}^{(\alpha,\beta)}(x), \quad f^{-1}(y) \ne \phi \ ; \ \inf \ \mu'^{(\alpha,\beta)}_{P_x}(x) \ , \quad f^{-1}(y) \ne \phi \\ \qquad\qquad\qquad\qquad 1 \ , \quad otherwise \\ 0 \ , \quad otherwise \end{bmatrix}$$

$$= \begin{cases} (\alpha,\beta) & if \quad y = f(x) \\ (0, \ 1) & if \quad y \ne f(x) \end{cases}$$

$$= P_{f(x)}^{(\alpha,\beta)} \ (y)$$

$$\Rightarrow f\left(P_x^{(\alpha,\beta)}\right) = P_{f(x)}^{(\alpha,\beta)}$$

(ii) we have

$$f\left\{ \left(P_x^{(\alpha,\beta)}\right)^C \right\}(y) = f\left\{P_x^{(1-\alpha,1-\beta)}\right\}(y)$$

$$= \begin{bmatrix} \sup\limits_{x\in f^{-}(y)} P_x^{(1-\alpha,1-\beta)}(x) \quad f^{-1}(y) \ne \phi \ ; \ \inf \ \mu'_{P_x}{}^{(1-\alpha,1-\beta)}(x) \ , \quad f^{-1}(y) \ne \phi \\ \qquad\qquad\qquad\qquad 1 \ , \quad otherwise \\ 0 \ , \quad otherwise \end{bmatrix}$$

$$= \begin{cases} (1-\alpha,\ 1-\beta) & if \quad y = f(x) \\ (0,\ 1) & if \quad y \neq f(x) \end{cases}$$

$$= f\left(P_x^{(1-\alpha,1-\beta)}\right)(y)$$

$$\left\{f\left(P_x^{(\alpha,\beta)}\right)\right\}^C(y)$$

$$so, \quad f\left\{ (P_x^{(\alpha,\beta)})^C \right\} = \left\{ f\left(P_x^{(\alpha,\beta)}\right) \right\}^C$$

Theorem (4.10) : β is a base for an IFTP (X, τ) iff for all $\tilde{G} \in \tau$ and for any IFP P in $\tilde{G}$, $\exists$ $B \in \beta$ such that $P \in B \subseteq \tilde{G}$.

Proof : Let β be a base for an IFTS (X, τ)

$\therefore \tilde{G} \in \tau \Rightarrow \tilde{G}$ is the union of some members of β

Let $\tilde{G} \in \tau$ be such that $P_x^{(\alpha,\beta)} \in \tilde{G}$

Now $\tilde{G} = \bigcup_{i \in I} \{ Bi \ : \ Bi \in \beta \}$

So, $P_x^{(\alpha,\beta)} \in \tilde{G} \Rightarrow P_x^{(\alpha,\beta)} \in \bigcup_{i \in I} \{ Bi \ : \ Bi \in \beta \}$

$\Rightarrow P_x^{(\alpha,\beta)} \in B_x \subseteq \tilde{G}$ for some B_x

Conversely suppose that for each $\tilde{G} \in \tau$ and for each $P_x^{(\alpha,\beta)} \in \tilde{G}$, $\exists$ B_x such that $P_x^{(\alpha,\beta)} \in B_x \subseteq \tilde{G}$.

To prove that $\tilde{G}$ can be written as a union of membersof β.

Consider any arbitrary $P_x^{(\alpha,\beta)} \in \tilde{G}$. Then by hypothesis $\exists$ $B_x \in \beta$ such that

$P_x^{(\alpha,\beta)} \in B_x \subseteq \tilde{G}$

$\Rightarrow \tilde{G} \subseteq \bigcup_{P_x(\alpha,\beta)} B(x)$

But $B_x \subseteq \tilde{G}$ $\forall$ $P_x^{(\alpha,\beta)} \in \tilde{G}$. So, $\tilde{G} = \bigcup_{P_x(\alpha,\beta) \in G} B$

$\Rightarrow \beta$ is a base for (X, τ).

5. Closure and Interior of IFS :

Definition (5.1) : Let (X, τ) be an IFTS and $\tilde{A} = \left\{ \left(x, \quad \mu_{\tilde{A}}(x), \quad \mu'_{\tilde{A}}(x) \right) \right\}$ be an IFS in X.

Then the intuitionistic fuzzy closure and the intuitionistic fuzzy interior of $\tilde{A}$ are denoted and defined by

$$\overline{\tilde{A}} = \cap \{ \ \tilde{K} \ : \ \tilde{K} \text{ is an intuitionistic fuzzy closed set in X and } \tilde{A} \subseteq \tilde{K} \ \}$$

$$\left(\tilde{A}\right)^{o} = \cup \{ \ \tilde{G} : \tilde{G} \in \tau \text{ and } \tilde{G} \subseteq \tilde{A} \ \}$$

Example (5.2) : Let X = { a, b, c } and $\tilde{A}$, $\tilde{B}$ be two IFSs in X given by

$\tilde{A}$ = { (a, .5, .2), (b, .5, .1), (c, .8, .2) }

$\tilde{B}$ = { (a, .2, .6), (b, .1, .6), (c, .1, .9) }

Then $\tau = \left\{ \tilde{0}, \quad \tilde{A}, \quad \tilde{B}, \quad \tilde{A} \cup \tilde{B}, \quad \tilde{1} \right\}$ is an IFT on X because $\tilde{A} \cup \tilde{B} = \tilde{A} \in \tau$, $\tilde{A} \cap \tilde{B} = \tilde{B} \in \tau$

Now it can be easily seen that $\overline{\tilde{A}} = \left(\tilde{B}\right)^{c}$

Because $\left(\tilde{B}\right)^{c}$ = { (a, .6, .2), (b, .6, .1), (c, .9, .1) } = $\tilde{K}$ (Say)

Clearly $\tilde{A} \subseteq \tilde{K}$

Because .5 $\langle$ .6, .2=.2, .1=.1, .8 $\langle$ .9, .2 $\rangle$ .1, [def. (2.5)]

Hence $\overline{\tilde{A}} = \tilde{K} = \left(\tilde{B}\right)^{c}$

Since $\tilde{B} \in \tau$, So $\overline{\tilde{A}}$ is a closed set

Similarly it can be shown that $\overline{\tilde{B}} = \left(\tilde{A}\right)^{C}$

$$\overline{\tilde{A} \cup \tilde{B}} = \tilde{1}, \ \left\{ \left(\tilde{A}\right)^{C} \right\}^{O} = \tilde{B}, \ \left\{ \left(\tilde{B}\right)^{C} \right\}^{O} = \tilde{A} \text{ and } \left(\tilde{A} \cup \tilde{B}\right)^{O} = \tilde{O}$$

6. Neighbourhood:

Definition (6.1) : An IFP $P_x^{(\alpha,\beta)}$ is said to be quasi coincident with an IFS $\tilde{A}$ denoted by $P_x^{(\alpha,\beta)} q \ \tilde{A} \ \textit{iff} \quad \alpha \ \rangle \ 1 - \mu_{\tilde{A}}(x)$

$$\beta \langle 1-\mu'_{\tilde{A}}(x) \qquad \forall \quad x \in X$$

Example (6.2) : Let X = { a, b } and $\tau = \{ \tilde{0}, \ \tilde{G}_1, \ \tilde{G}_2, \ \tilde{1} \}$ be an IFT on X, where

$\tilde{G}_1$ = { (a, .3, .6), (b, .2, .7) } and $\tilde{G}_2$ = { (a, .5, .3), (b, .6, .4) } are IFSs in X,

Then the IFP $P_x^{(.9,\ .1)}$ is quasi coincident with the IFS $\tilde{G}_1$

Because 1 - .3 = .7, 1 - .6 = .4

1 - .2 = .8, 1 - .7 = .3

So $\alpha \rangle 1-\mu_{\tilde{G}_1}(x)$

` $\beta \langle 1-\mu'_{\tilde{G}_1}(x)$ is satisfied condition.

Hence $P_x^{(.9,\ .1)}\ q\ \tilde{G}_1$

Proposition (6.3) : $P_x^{(\alpha,\beta)} \in \tilde{A}$ *iff* $P_x^{(\alpha,\beta)}$ is not quasi coincident with $(\tilde{A})^C$

Proof :Let $P_x^{(\alpha,\beta)} \in \tilde{A} \Leftrightarrow \alpha \le \mu_{\tilde{A}}(x) \quad and \quad \beta \ge \mu'_{\tilde{A}}(x)$ [Def. (4.7)]

$\Leftrightarrow 1-\alpha \ge 1-\mu_{\tilde{A}}(x) = \mu_{(\tilde{A})^C}(x)$

and $1-\beta \le 1-\mu'_{\tilde{A}}(x) = \mu'_{(\tilde{A})^C}(x)$

$\Leftrightarrow 1-\mu_{\tilde{A}^C}(x) \ge \alpha$ and $1-\mu'_{\tilde{A}^C}(x) \le \beta$

$\Leftrightarrow \alpha \le 1-\mu_{\tilde{A}^C}(x)$ and $\beta \ge 1-\mu'_{\tilde{A}^C}(x)$

$\Leftrightarrow P_x(\alpha,\beta)\ \not{q}\ (\tilde{A})^C$

Definition (6.4) : An IFS $\tilde{A}$ in (X, τ) is called a neighbourhood of IFP $P_x^{(\alpha,\beta)}$ *iff* $\exists$ a $\tilde{B} \in \tau$ such that $P_x^{(\alpha,\beta)} \in \tilde{B} \subseteq \tilde{A}$

A neighbourhood $\tilde{A}$ is intuitionistic fuzzy open *iff* $\tilde{A}$ is intuitionistic fuzzy open.

Definition (6.5) : An IFS $\tilde{A}$ in (X, τ) is called a Q-neighbourhood of IFP $P_x^{(\alpha,\beta)}$ *iff* $\exists$ a $\tilde{B} \in \tau$ such that $P_x^{(\alpha,\beta)}\ q\ \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$.

In the above example (6.2), $\tilde{G}_2$ is a Q-neighbourhood of the IFP $P_x^{(.9,\ .1)}$, because $\exists$ $\tilde{G}_1 \in \tau$ such that $P_x^{(.9,\ .1)}\ q\ \tilde{G}_1$ and $\tilde{G}_1 \subseteq \tilde{G}_2$

as $\mu_{\tilde{G}_1}(x) \langle\ \mu_{\tilde{G}_2}(x),\ \mu'_{\tilde{G}_1}(x) \rangle\ \mu'_{\tilde{G}_2}(x) \quad \forall \quad x \in X$ [Def. (2.5)]

Definition (6.6) : An IFP 'e' is called an adherent point of an IFS $\tilde{A}$ *iff* every Q-neighbourhood of 'e' is quasi coincident with A.

Again 'e' is called an accumulation point of an IFS $\tilde{A}$ *iff* 'e' is an adherent point of $\tilde{A}$ and every Q-neighbourhood of 'e' and $\tilde{A}$ are quasi coincident at some point other thansupp(e) whenever $e \in \tilde{A}$ '

The union of all accumulation points of $\tilde{A}$ is called the derived set of $\tilde{A}$ denoted by $D(\tilde{A})$.

It is evident that $D(\tilde{A}) \subset \overline{\tilde{A}}$.

Theorem (6.7) : In an IFTS, $\overline{\tilde{A}} = \tilde{A} \cup D(\tilde{A})$

Proof : Let S = { e : e is an adherent point of $\tilde{A}$ }

Then $\overline{\tilde{A}} = \cup S$

On the other hand $e \in S \Rightarrow e \in D(\tilde{A})$

$\therefore\ \overline{\tilde{A}} = \cup S \subset \tilde{A} \cup D(\tilde{A})$

The converse part follows directly from the fact that $D(\tilde{A}) \subset \overline{\tilde{A}}$ and $\tilde{A} \subset \overline{\tilde{A}}$

$\therefore\ \tilde{A} \cup D(\tilde{A}) \subset \overline{\tilde{A}}$

So $\overline{\tilde{A}} = \tilde{A} \cup D(\tilde{A})$

Theorem (6.8) : An IFS $\tilde{A}$ is closed iff $\tilde{A}$ contains all the accumulation point of $\tilde{A}$.

Proof : we know that

$\overline{\tilde{A}} = \tilde{A} \cup D(\tilde{A})$

Again an IFS $\tilde{A}$ is closed if $\overline{\tilde{A}} = \tilde{A}$

$$\therefore \tilde{A} = \overline{\tilde{A}} = \tilde{A} \cup D(\tilde{A})$$

$$\Rightarrow D(\tilde{A}) \subseteq \tilde{A}$$

Conversely let $\tilde{A}$ contain all accumulation point of $\tilde{A}$

$$\Rightarrow D(\tilde{A}) \subseteq \tilde{A}$$

And hence $\overline{\tilde{A}} = \tilde{A} \cup D(\tilde{A})$

$$\Rightarrow \overline{\tilde{A}} = \tilde{A}$$

$\Rightarrow \tilde{A}$ is closed.

7. Conclusion :

With this new intuitionistic fuzzy topology, one can study compactness, connected ness and separation axioms.

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